Implications of Parameter Uncertainty for the CBN's Monetary Policy*

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ABSTRACT

This paper examines the empirical importance of multiplicative parameter uncertainty on the conduct of the Central Bank of Nigeria's monetary policy over the period 1980Q1–2015Q1. Theoretically, the certainty equivalence principle indicates that the optimal policy is not affected by the degree of uncertainty called "additive". However, the "Brainard conservatism principle" states that under uncertainty about the transmission mechanism, monetary policy should be less aggressive than in certainty universe. We show in this study that the Brainard principle can be challenged not only by the choice of the model used but also by the preferences of the central bank. Using the framework of a parameterized model with parsimony of IS curve-Phillips curve-type and a simple rule, and the linear quadratic stochastic control approach by introduction the variance of the estimated parameters in the optimal control theory the results yield that the central banker are always very cautious when they have an inflation and output stabilization objective. However, when they are concerned to smooth interest rate, their behavior becomes more aggressive, with a degree of aggressiveness that depends in part on the objective of interest rate smoothing.

Keywords: Optimal monetary policy, Parameter uncertainty, Brainard conservatism principle, Interest rate smoothing, Nigeria.
JEL classification: E43, E52, E58.

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1. INTRODUCTION

Uncertainty is constantly present in economics and monetary authorities should still face uncertainty (Onatski and Williams 2003). For Greenspan (2003), the former Federal Reserve Chairman, "Uncertainty is not just an important feature of the monetary policy landscape, it is the defining characteristic of that landscape" and for the former European Central Bank Governor, Trichet (2011), further adds that "Operating in an uncertain environment is common business for central banks."

For most developing countries, the key risks are the same due to the similarity of economic and social conditions. Therefore, the conduct of monetary policy in Nigeria at its core involves crucial elements of risk management, a process that requires an understanding of the many sources of risk and uncertainty that policymakers face and the quantifying of those risks when possible. It also entails devising, in light of those risks, a strategy for policy directed at maximizing the probabilities of achieving over time their goal of price stability and the maximum sustainable economic growth that they associate with it.

The primary goal of monetary policy in Nigeria has been the maintenance of domestic price and exchange rate stability since it is critical for the attainment of sustainable economic growth and external sector viability (Sanusi, 2002). Over the years, the objectives of monetary policy in Nigeria have remained the attainment of internal and external balance of payments. However, emphasis on techniques/instruments to achieve those objectives have changed over the years. There have been two major phases in the pursuit of monetary policy, namely, before and after 1986. The first phase placed emphasis on direct monetary controls, while the second relies on market mechanisms.

The CBN's monetary policy in 2008, for example, continued to be shaped by developments in the global and domestic economic and financial environment. At the global level, the key influences were: increased monetary policy divergence among the advanced economies; continued uncertainties surrounding the BREXIT negotiations and sustained monetary policy normalization in the US as the Fed hiked its interest rate and gave forward guidance of more, with implications for capital reversals from the emerging markets and developing economies. Others included the U.S withdrawal from the Iranian nuclear deal, the emerging trade tensions between the US and other major world economies as well as pockets of geopolitical tensions, the terrorist attacks carried out by Boko-Haram. These, notwithstanding, the global economy continued on the path of recovery, stemming from the strengthening of domestic investment demand and relatively easier financing conditions in the advanced economies, as well as the sustained recovery in oil and other commodity prices, amid limited spillovers of trade tensions to market sentiments. Monetary policy in 2018, was informed by key considerations which included; the slow output recovery; high but moderating inflation rate which remained above the Bank's target range; continuing liquidity surfeit in the banking system; weak macro-prudential indicators; growing sovereign debt and low fiscal buffers. These developments and the need to achieve the Bank's mandate of price and exchange rate stability provided the basis for the sustenance of the tight monetary policy stance during the year.

According to Uchendu (2009), the conduct of monetary policy in Nigeria confronts three kinds of uncertainty principally uncertainty about monetary and other data, uncertainty about the transmission mechanism of monetary policy, and uncertainty about the fiscal policy outlook. Sure enough, about the transmission mechanism of monetary policy, limited knowledge is a common feature in developing countries. For Nigeria knowledge of the relative strengths of the transmission channels of monetary policy is still at the rudimentary level. The transmission mechanism of monetary policy is not fixed, that is to say, changes over time. The Monetary policy is formulated with some assumptions about the path (and/or strength of the path) through which policy impacts on the economy. The uncertainties add to complications of monetary management. In fact, to deal with uncertainty about the transmission mechanism and underlying macro relationships, Central Bank of Nigeria (CBN) encourages and sponsors research and utilizes the outputs of those studies in making an opinion about monetary policy transmission on a continuous basis, monetary policymakers at the Bank learn extensively Nigeria's and other countries' experiences in making assumptions about the transmission path of monetary policy, overtime experiences are formalized through independent as well as Bank-led studies and knowledge sharing, development of institutional memory-efforts in this direction has received a boost in recent years as part of the internal reform initiatives of the Bank (Uchendu, 2009). What then are the implications of this largely irreducible multiplicative parameter uncertainty for the conduct of the CBN's monetary policy?

In the absence of consensus both on the existence and the size of the 'Brainard effect' which states that under uncertainty about the transmission mechanism, monetary policy should be less aggressive than in certainty universe, this study attempts to reconcile the conservatism principle with the reality of the Nigerian economy as reflected daily the evolution of the CBN's monetary policy rates. Thus, the aims of this study is to analyze the implication of parameter uncertainty on the CBN's central banker behavior.

The paper is organized as follows: the next section presents the literature review, Section 3 presents the model of the study; Section 4 presents the influence of parameter estimation uncertainty while, Section 5 discusses the effects of multiplicative parameter uncertainty on optimal policy and finally Section 6 concludes the paper.

2 LITERATURE REVIEW

2.1 Theoretical literature

The monetary policy responses to uncertainty traditionally presented in the literature can be summed up in two main lessons : first, the principle of certainty equivalence, due to Theil (1958), indicates that the optimal policy is not affected by the magnitude so-called "*additive*" uncertainty, that is those relating to temporary shocks; and second instead, the cautious principle, associated with the name Brainard (1967), states that uncertainty about the transmission mechanisms, monetary policy needs to be less aggressive than in certainty universe. Indeed, the uncertainty about shocks and measurement errors of the variables of interest of the monetary authorities is qualified as additive uncertainty.

According to Theil (1958), the principle of certainty equivalence is the most important result in the presence of this type of uncertainty. According to this principle of certainty equivalent, under an additive uncertainty, the optimal policy is the same as in the absence of uncertainty. In other words, monetary policy must respond to the best estimate of variables exactly as it would in the presence of perfectly measured variables (Poole, 1970).

According to the original analysis of William Brainard (1967), it is widely accepted that policymakers facing uncertainty about the structure of the economy should be more cautious when implementing policy than if acting under complete certainty. The attractiveness of this result, named the "*Brainard conservatism principle*" by Alan Blinder (1997 and 1998), lies in both the simplicity of the original argument and in the underlying intuition: when you are uncertain about the effects of policy, it makes sense for policymakers to move more cautiously in the response to economic shocks. This principle states that under uncertainty on the key parameters describing the transmission mechanisms, the monetary policy must be less aggressive than in the certain universe. According to this principle, the uncertainty on the parameters justifies the adoption of a conservative monetary policy, that is, acting with precaution during the adjustments of the monetary policy instrument.

It is important to make difference between risk and uncertainty. Indeed, the importance of risks and uncertainty for economic analysis was initially raised by Knight (1921). "*Risk*" refers to situations in which the decision-maker can assign probabilities to different events to which he is likely to be confronted; "*Uncertainty*" is applied to the situations where this random character cannot be expressed in terms of probabilities. So, Knight's uncertainty describes a risk that is not measurable. This theory is often seen as the link between that of Adam Smith (idea of risk) and that of John Maynard Keynes (notion of uncertainty).

2.2 Empirical literature

To test the veracity of the "*Brainard conservatism principle*", recent studies (Sack 1998 and 1999; Peersman and Smets 1999; Martin and Salmon 1999; Söderström 1999 and 2002; Estrella and Mishkin, 1999; Wieland 1998; Srour 1999; Rudebusch 2001; Shuetrim and Thompson 1999; Orphanides et *al*. 2000; Ehrmann and Smets 2003; Sahuc 2005; among others) have used Bayesian methods associate with optimal control theory to determine the optimal monetary policy that minimizes the excepted loss function, given a prior distribution on some uncertain parameters and analyzed the empirical importance of uncertainty about the parameters and the scope of the caution principle. The results obtained in the literature depend on the type of models used. Studies using a parameterized model with parsimony of IS curve-Phillips curve-type and a simple rule (Estrella and Mishkin 1999; Rudebusch 2001; Srour 1999; Shuetrim and Thompson 1999; Sahuc 2005; among others) frequently conclude that uncertainty about the parameters is not an important source of mitigation of the monetary policy responses. However, the studies based on vector autoregressive models (VAR) (Sack 1998 and 1999; Martin and Salmon 1999; Söderström 1999; among others) broadly support the caution principle. These authors attempt to quantify the effects of attenuation within VAR models and using unconstrained policy rules. At each period of the sample, the optimal interest rate can be compared with the observed interest rate.

Others authors as Giannoni (2002) and Söderström (2002) have presented evidence that supports an aggressive reaction of monetary policy under uncertainty. Giannoni (2002) for example develops a model based on a property of zero-sum two-player games to determine a robust optimal monetary policy rules particularly in a situation of uncertainties about the parameters of the structural model. Then, the author applied it to an optimal Taylor rules in a simple forward-looking macroeconomic model. The results, contrary to the common belief that monetary policy should be less responsive in case of parameter uncertainty, show stronger reaction of nominal interest rate to fluctuations in the rate of inflation and output gap as against the period of certainty.

Cateau (2007) finds that, if the central bank cares strongly enough about stabilizing the output gap, this aversion generates significant declines in the coefficients of the Taylor rule, even if the bank's loss function assigns little weight to reducing interest rate variability. The author also finds that an aversion to model and data-parameter uncertainty can yield an optimal Taylor rule that matches the empirical Taylor rule. Under some conditions, a small degree of aversion is enough to match the historical rule.

Using a stylized macroeconomic model on US data to examine the implications of uncertainty about the effects of monetary policy for optimal monetary policy with an application to the current situation, Williams (2013) highlights three important insights for monetary policy under uncertainty. First, even in the presence of considerable uncertainty about the effects of monetary policy, the optimal policy nevertheless responds strongly to shocks: uncertainty does not imply inaction. Second, one cannot simply look at point forecasts and judge whether policy is optimal. Indeed, once one recognizes uncertainty, some moderation in monetary policy may well be optimal. Third, in the context of multiple policy instruments, the optimal strategy is to rely on the instrument associated with the least uncertainty and use alternative, more uncertain instruments only when the least uncertain instrument is employed to its fullest extent possible.

By using a parameterized model with parsimony of IS curve-Phillips curve-type and a simple rule in the WAEMU zone and the linear quadratic stochastic control approach by introduction the variance covariance matrix of the estimated parameters in the optimal control theory, Nantob (2015) through the optimal monetary policy rule concludes that the policymakers are always very cautious when they have an inflation and output gap stabilization objective. However, when they are concerned to smooth interest rate, their behavior becomes distinctly more aggressive.

To study the impact of uncertainty about the true state of the economy on monetary policy in South Africa since the adoption of inflation targeting, Naraidoo and Raputsoane (2015) indicate that the effect of uncertainty on the interest rates has led to a more cautious monetary policy stance by the monetary authorities consistent with a large body of literature that recognizes that an excessively activist policy can increase economic instability.

Exploring the role of monetary policy uncertainty on the strategic interaction between fiscal and monetary policies, Giovanni and Giuli (2010) highlight that monetary uncertainty and fiscal uncertainty are not symmetric. The monetary uncertainty may induce both more and less aggressive effects on the final outcomes according to the kind of existing interaction between the government and the central bank. The multiplicative uncertainty implies an endogenous Phillips relationship between inflation and output, which does not emerge under fiscal uncertainty.

Gnabo and Moccero (2015) contribute to the empirical literature on the risk-management approach to monetary policy by estimating regime switching models where the strength of the response of monetary policy to macroeconomic conditions depends on the level of risk associated with the inflation outlook and risk in financial markets. Using quarterly data for the Greenspan period they find that: first, risk in the inflation outlook and in financial markets are a more powerful driver of monetary policy regime changes than variables typically suggested in the literature, such as the level of inflation and the output gap; second, estimation of regime switching models shows that the response of the US FED to the inflation outlook is invariant across policy regimes; third, however, in periods of high economic risk monetary policy tends to respond more aggressively to the output gap and the degree of inertia tends to be lower than in normal circumstances; and fourth, the US FED is estimated to have responded aggressively to the output gap in the late 1980s and beginning of the 1990s, and in the late 1990s and early 2000s. These results are consistent with Mishkin (2008)'s view that in periods of high economic risk monetary authorities should respond aggressively to changes in macroeconomic conditions while the degree of inertia should be lower than in normal circumstances.

To analyze the uncertainty about the structure of the economy in the WAEMU, Nantob (2014) used the DSGE model which is a good tool to investigate the responses of optimal monetary policy under commitment on a policy rule (Ramsey policy) or a simple rule, both under certainty equivalence and parameter uncertainty. The author found that under commitment on a policy rule or a simple rule, optimal responses of welfare, investment, real value of capital, capital stock, inflation rate, real wages, labor hours, capital utilization rate and production following shocks of preference, rental prices of capital, technology, prices mark-up, wages mark-up, labor supply and inflation objective are relatively flat under uncertainty and optimal policies are relatively aggressive. Furthermore, the study found that under the same optimal monetary regimes, optimal responses of the same variables following now shocks of investment, wages mark-up and government expenditure are relatively persistent under parameter uncertainty and optimal policies are relatively cautious. Kurozumi (2010) studies the optimal monetary policy under uncertainty about fundamental parameters of a DSGE model and finds the conditions under which optimal discretionary policy responds to shocks more aggressively than in the absence of the uncertainty. These conditions depend crucially on the persistence of shocks and the magnitude of policy multipliers. To obtain the conditions, taking proper account of uncertainty about the transmission of shocks and about the welfare loss function is of crucial importance. Edge et *al*. (2010) examines welfare-maximizing monetary policy in an estimated micro-founded general equilibrium model of the US economy where the policymaker faces uncertainty about model parameters and they find that optimal Taylor rules under parameter uncertainty respond less to the output gap and more to price inflation than would be optimal absent parameter uncertainty. They also show that policy rules that focus solely on stabilizing wages and prices yield welfare outcomes very close to the first-best. Using a micro-founded macroeconometric, Sala et *al*. (2008) find that the estimated natural rate of unemployment in US is consistent with the NBER description of the U.S. business cycle, and that the inflation/unemployment trade-off facing monetary policymakers is quantitatively important. They also show that parameter uncertainty has a limited effect on the performance or design of monetary policy, while natural rate uncertainty has more sizeable effects. Nevertheless, policy rules that respond to the output or unemployment gaps are more efficient than rules responding to output or unemployment growth rates, also in the presence of uncertainty about the natural rates.

By showing that dispersion-based uncertainty about the future course of monetary policy is the single most important determinant of Treasury bond volatility across all maturities, Arnold and Vrugt (2010) find that the link between Treasury bond volatility and uncertainty about macroeconomic variables is much stronger than for the more traditional time series measures of macroeconomic volatility and adds beyond the information contained in lagged bond market volatility. Uncertainty about monetary policy subsumes the uncertainty about future inflation (consumer price index and the deflator) and economic activity (unemployment, real and nominal gross domestic product and industrial production). In addition, causality clearly runs one way: from monetary policy uncertainty to Treasury bond volatility.

Bartolomeo et *al*. (2009) extends a well-known macroeconomic stabilization game between monetary and fiscal authorities developed by Dixit and Lambertini (2003) to multiplicative (policy) uncertainty and find that even if fiscal and monetary authorities share a common output and inflation target (i.e., the symbiosis assumption), the achievement of the common targets is no longer guaranteed; under multiplicative uncertainty, in fact, a time consistency problem arises unless policymakers' output target is equal to the natural level.

Rudebusch and Wu (2008) develop and estimate a macro-finance model that combines a canonical affine no-arbitrage finance specification of the term structure of interest rates with standard macroeconomic aggregate relationships for output and inflation. Based on this combination of yield curve and macroeconomic structure and data, they obtain several results: first, the latent term structure factors from no-arbitrage finance models appear to have important macroeconomic and monetary policy underpinnings, second, there is no evidence of a slow partial adjustment of the policy interest rate by the central bank, and third both forward-looking and backward-looking elements play roles in macroeconomic dynamics.

Svensson and Williams (2008) use a Markov Jump Linear-Quadratic (MJLQ) approach to design an optimal monetary policy instruments under uncertainty. Various discrete models were used to estimate different types of uncertainties that policy makers contend with. With Markov chain and mode-dependent linear-quadratic approximations of the underlying model, the authors apply algorithms to analyze effects of uncertainties as well as potential gains in a New Keynesian Phillips curve model. The results show that new initiatives by central banks significantly affect losses.

By studying the cost channel of monetary transmission, Tillmann (2009) shown that the Brainard (1967) principle of cautious policy in the face of uncertainty continues to hold in both a Bayesian and a minmax framework. Drissi (2014) study the robust monetary policy of Tunisia in an uncertain economic environment and found that the uncertainties of the structural parameters affect the dynamic solutions for the economy, but also on the objective functions of the central bank. The caution of the central bank increased with the weights carried by the interest rate in the loss function. By using robust control approach to study how a central bank in an economy with imperfect interest rate pass-through conducts monetary policy if it fears that its model could be misspecified, Gerke and Hammermann (2011) find that the effects of the central bank's concern for robustness can be summarised as follows : first, depending on the shock, robust optimal monetary policy under commitment responds either more cautiously or more aggressively; second, such robustness comes at a cost: the central bank dampens volatility in the inflation rate preemptively, but accepts higher volatility in the output gap and the loan rate and, third, if the central bank faces uncertainty only in the IS equation or the loan rate equation, the robust policy shifts its concern for stabilisation away from inflation.

By determining a robust optimal policy rule in a forward-looking model, under conditions of policy maker's uncertainty about model parameters and shock processes, Giannoni (2007) finds that an optimal policy rule requires a robust reaction of the interest rate to movements in both inflation and output gaps as compared to the case when policy makers are certain about model parameters and shock processes, and concludes, therefore, that although the parameter uncertainty is not necessary for a trivial response of monetary policy to distress but it is capable of enlarging the degree of apathy required by optimal monetary policy.

Yilmaz et *al*. (2009) analyze the implied United States Federal Reserve Bank (FED) policy behavior under multiplicative model and shock uncertainty, defined through performance objectives, cases by using historical data. Using robust system theory frameworks to study empirically the characteristics of the FED short-term interest rate-inflation dynamics under different circumstances by using a single-input single-output model, they demonstrate that the historical FED actions were conservative under model and shock uncertainty. Olalla and Gómez (2011) seek to explain the recent behaviour of the two main central banks in the recent financial crisis, applying a robust control tool through a Neo-Keynesian monetary policy model. They mainly find that the different results depend on the behaviour of the law of motion of the state variables, specifically the shadow prices that influence the private sector's expectations.

Mendes et *al.* (2017) summarize and compare the main results that have emerged in the literature on optimal monetary policy under uncertainty with actual central bank behaviour. Their analysis of relevant examples of Bank of Canada policy confirms that uncertainty does have profound effects on monetary policy, leading policy-makers to deviate substantially from what a typical monetary policy rule would suggest.

It appears from this literature review that there is no consensus on the behavior of the central banker in the face of uncertainty. Moreover, to our knowledge, there is no studies on the CBN's monetary policy management under uncertainty particularly under multiplicative uncertainty. This is the objective of this study.

3. THE MODEL

3.1 Theoretical framework of the conduct of monetary policy under parameter uncertainty

Brainard (1967) indicates that the certain equivalent is not always verified for the complex specifications of uncertainty. More specifically, when it is about the uncertainty surrounding the parameters of the model, the central bank will not behave as if uncertainty did not exist. This result has been described by Blinder (1997; 1998) as "*Brainard conservatism principle*". The difference is that in this uncertainty surrounding the parameters, the uncertainty is multiplicative rather than additive: the more the policy is used the more this uncertainty is multiplicative in the system. To see how change the nature of the optimal policy, consider the monetary transmission mechanism under uncertainty about the parameters.

Equation (1) describes the mechanism of monetary transmission in which inflation p is determined by the interest rate i at through the knowledge of the coefficient b where b<0. u is an error term ($u \sim i.i.d$) of zero mean and variance equal to s_u^2 .

$$p = bi + u \qquad (1)$$

où $b \to \left(\hat{b}, s_b^2\right)$.

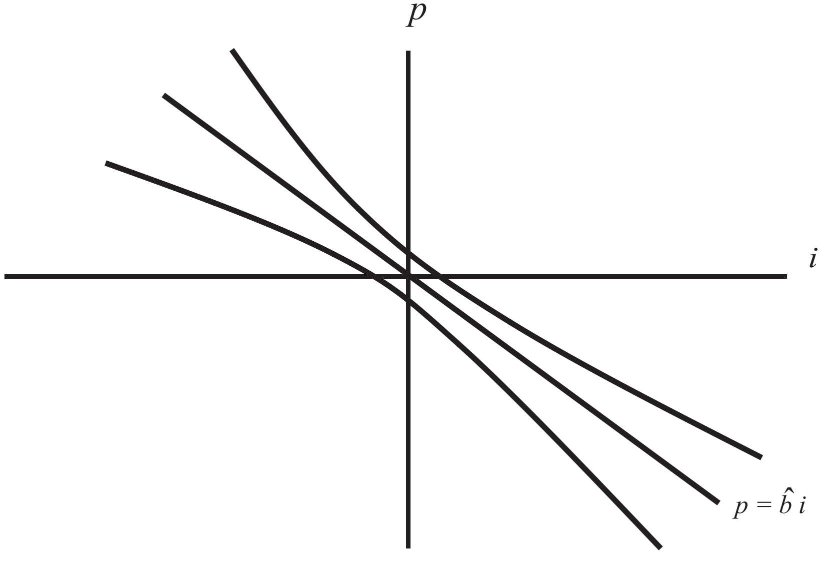
There is uncertainty about parameter b in equation (1). However, although central bank does not know the precise value of b, it knows its distribution (that is the central bank knows the mean of b and the variance s_b^2).

There are several reasons why a reasonable description of the central bank behavior on the policy monetary transmission mechanisms is needed. Maybe the central bank has low information on how its policy transmission mechanisms works. Alternatively, there may be fundamental uncertainties in the monetary policy transmission which in prelude have never been able to predict with certainty what is the effect of the interest rate on inflation. The structure of the stylized economy (1) appears in figure (1). The central line shows the relationship $p = bi$, which is important in the forecasts. The curves are confidence bands showing the range of inflation that is expected to give the interest rates.

Figure (1) below shows how the uncertainty on parameter b is multiplicative. When interest rates evolve far from zero, it becomes more and more difficult to predict the level of inflation. Uncertainty could be minimized with the zero interest rate for which the only uncertainty is additive because of the error term, but whose expected inflation would be zero and not equal to the target. Mathematically, the expected loss with uncertainty about the parameters is given by:

$$\begin{aligned} L^e &= E\left[\left(bi + u - p^*\right)^2\right] \\ &= E\left(b^2\right)i^2 + E\left(u^2\right) + p^{*2} + 2E\left(bu\right)i - 2E\left(b\right)ip^* - 2p^*E\left(u\right) \end{aligned}$$

<u>Figure 1</u>: Structural uncertainty



Similarly, the definition of *u* as a random error term implies $\mathrm{E}(u^2)$ and s_u^2 and $\mathrm{E}(u)$ = 0, (that is). The mean of □ is given by $\mathrm{E}(\hat{b}) = \mathrm{b}$ and the variance of $\hat{b}$ can be written as $s_u^2 = \mathrm{E}\left(b - \hat{b}\right)^2 = \mathrm{E}\left(b^2\right) - \mathrm{E}\left(\hat{b}^2\right)$, which gives the expression of $\mathrm{E}\left(b^2\right)$. By making moreover a simplistic assumption that the uncertainties on *b* and *u* have no relation, in other words E (*bu*) = 0, the expected loss can be described by equation (2):

$$\mathrm{L}^e = s_b^2 i^2 + s_u^2 + \left(bi - p^*\right) \qquad (2)$$

The optimal policy (3) is deduced by derivation of equation (2) from the interest rate *i*:

$$i = \frac{bp^*}{b^2 + s_b^2} \qquad (3)$$

The presence of parameter uncertainty means that the optimal policy is more cautious. $\left|i^{Brainard}\right| = \left|i^{Certainty\ Equivalence}\right|$ implies that interest rates are closer to zero under Brainard's policy than under the certainty equivalent. The reason is that the supplementary caution reduces the amount of uncertainty that politics introduces into the system. At the extreme, when $s_b^2 \rightarrow ¥$ and the uncertainty about the parameter becomes infinite, the optimal policy does nothing and sets the interest rate to zero, $\mathrm{i} \rightarrow 0$. In contrast, the uncertainty on the parameter disappears when $s_b^2 \rightarrow 0$ and the interest rate is set equal to that under the certain equivalent. This result is often referred to as the "*Brainard's conservatism*". The uncertainty on the parameter introduces a reason for caution in the optimal policy. Thus, the policy means that the central bank does not hope to reach its inflation target (that is $p^{e1}\ p^*$). The reason for this is that it aims to reach the exact target involving large potential losses, especially if the parameter *b* becomes large and the transmission mechanism is stronger than expected.

So, for this study, we suppose that face of the multiplicative uncertainty, the CBN's optimal monetary policy is less aggressive.

3.2. Setup

This study looks at the uncertainty linked to parameters estimation in an empirical model in Nigeria allowing a more appropriate dynamics for quarterly data. The basic model is similar to many other models used for monetary policy analysis (Ball 1997; Cecchetti 1998; Taylor 1994; Wieland 1998; Svensson 1997a and 1997b; Ellingsen and Söderström 1999; Söderström 1999; Rudebusch and Svensson 1999; Estrella and Mishkin 1999; Rudebusch 2001; Srour 1999; Shuetrim and Thompson 1999; Le Bihan and Sahuc 2002; Nantob 2015, etc.) and includes two curves: the IS curve (Equation 1) and Phillips curve (Equation 2):

$$y_{t+1} = g_y y_t - g_r \left(i_t - p_t\right) + e^{y}_{t+1} \quad (4)$$

$$p_{t+1} = b_p p_t + b_y y_t + e^{p}_{t+1} \quad (5)$$

where p_t is the inflation rate at time t, y_t is the output gap (the output gap is the deviation of output from its potential level. It is calculated in the usual way by the difference between log of the real GDP and the potential GDP. It is determined by applying a Hodrick-Prescott filter, the smoothing parameter is set to 1600 as it is usually the case with quarterly data), i_t is the nominal interest rate (the nominal interest rate is linked to the real interest rate r_t, and to the expectations of future inflation, p_{t+1}, by the Fisher-type equation[2]: $r_i = i_i - \mathrm{E}_t p_{t+1}$), e^{y}_{t+1} and e^{p}_{t+1} are uncorrelated random shocks with zero means and the variances equal respectively to s_y^2 and s_p^2.

The principal characteristic of the model is that inflation and the output gap react with a lag of one period to the interest rate, which reflects the presence of a period of transmission of the monetary policy. To set ideas, the period here is a quarter. Because of this transmission delay and the presence of random shocks in the economy, policymakers have imperfect control of inflation.

The parameters of the model are stochastic, and therefore time-varying as in Söderström (2002). When the policymaker set its interest rate instrument at time t, it is assumed to know all realizations of the parameters up to and including period t, but it does not know their future realizations, and thus cannot be certain about the effects of policy on the economy. For simplicity, we assume that each parameter is given by a constant mean plus a random shock.

The high degree of abstraction is not in itself a sign of weakness. This methodological shortcut is founded on three arguments. On the one hand, it allows simplifying: sure enough, when the solutions of the more brief models with rational anticipations are complex and require habitually the recourse to numerical methods, the basic Keynesian models can be analyzed graphically and solved analytically. On the other hand, many characteristics of the global economy are in conformity with the microeconomic realities. Finally, it can be advantageous letting to the model his simplicity and his small size, in order to manipulate it more easily and to deep our comprehension of the fundamental economic mechanisms. This type of model permits thus to introduce easily an additive uncertainty by adding hazards at the level of each equations (or variables) or a multiplicative parameter uncertainty supposing a distribution on the parameters of the model. Since the mechanisms are reduced to their

2 - In order to not add uncertainty to private agent's expectations, we assume that private agents and the central bank have the same expectations.

more simple equipment, it is possible to study with precision the effect of a type of uncertainty.

3.3. Optimal policy

To determine the optimal time path of interest rates i_t, we assume that the objective of central bank is to minimize a quadratic function of the deviation of interest variables which here are inflation rate p_t and output gap y_t, to their targets. Inflation target and output gap are assumed to be zero for simplification reasons. Central bank has therefore for aim to:

$$\underset{\{r_{t+t}\}_{t=0}^{¥}}{\mathrm{Min}}\ \mathrm{E}_t \sum_{t=0}^{¥} f^t\, \mathrm{L}\left[p_{t+t}, y_{t+t}\right] \qquad (6)$$

where f is the discount factor of the central bank to each period. Most studies define the objective of the central bank as the minimization of inflation variance or the sum of inflation and output gap variances; we have added h ere an interest rate smoothing objective.

$$\mathrm{L}\left[p_t, y_t, \left(i_t - i_{t-1}\right)\right] = p_t^2 + l_y y_t^2 + l_i \left(i_t - i_{t-1}\right)^2 \qquad (7)$$

$$\mathrm{L} = Y_t' K Y_t$$

where Y_t is the vector of the objective variables of the central bank:

$$Y_t = \left[y_t \quad p_t \quad i_t - i_{t-1}\right]'.$$

Thus :

$$Y_t = C_x x_t + C_i i_t$$

$$C_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, C_i = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \text{ and } K = \begin{bmatrix} l_y & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & l_i \end{bmatrix}$$

The parameters l_y and l ($l_y \geq$ 0 and $l_i \geq 0$ are respectively the weight of output gap and interest rate. The first order condition allows obtaining the optimal path of the interest rate. But, in order to simplify the analysis, we consider that monetary authorities control the short run real interest rate:

$$r_t = i_t - \mathrm{E}p_{t+1} \qquad (10)$$

This hypothesis is fairly restrictive: the inflation rate is predetermined, and a rule of real interest rate can be written as a rule of nominal rates. Moreover, the model is linear and the objective function quadratic, the optimal response is a linear function of the variables , p_t, y_t and i_t - i_{t-1}:

$$r_t = f\left[p_t, y_t, (i_t - i_{t-1})\right] \quad (11)$$

In the simplest case, when the parameters are non-stochastic, it is relatively easy to determine the analytical solution of the optimization problem.

To solve the central bank's optimization problem under multiplicative parameter uncertainty, we rewrite the model (4)–(5) under state spaces form as follow:

$$x_{t+1} = A_{t+1}x_t + B_{t+1}i_t + e_{t+1} \quad (12)$$

where $x_{t+1} = \left[y_{t+1} \;\; p_{t+1} \;\; i_t\right]'$ is a state vector, and $e_{t+1} = \left[e^y_{t+1} \;\; e^p_{t+1} \;\; 0\right]'$ is the vector of the structural shocks. The parameters matrices A_{t+1} and B_{t+1} are thus stochastic with means A and B and variance-covariance matrices Σ_A, Σ_B and Σ_{AB} (see Appendix A for the representation).

The objective function in this structure will be quadratic and the constraint linear, like this the value function will have the form:

$$J\left(x_{t+1}\right) = x_{t+1}'\tilde{V}x_{t+1} + \tilde{w} \quad (13)$$

where the matrix $\tilde{V}$ remains to be determined.

When the parameters are stochastic, the certainty equivalence does not prevail more, since the variance of the state vector x_{t+1} intervenes in the optimal policy. In this case, the expected value of the value function (13) is:

$$\mathrm{E}_t J\left(x_{t+1}\right) = \left(\mathrm{E}_t x_{t+1}\right)'\tilde{V}\left(\mathrm{E}_t x_{t+1}\right) + tr\left(\tilde{V}\Sigma_{t+1|t}\right) + \tilde{w} \quad (14)$$

where Σ_{t+1t} is the matrix of variance-covariance of x_{t+1} valued in t, and "*tr*" the trace operator. In consequence, the matrix of variance-covariance of x_{t+1} that integrate the variances of the parameters will affect the optimal policy.

Appendix B shows that the optimal interest rate is given by:

$$i_t = \tilde{f}x_t \quad (15)$$

where the decision vector $\tilde{f}$ is given by:

$$\tilde{f} = -\left[R + B'\left(V + V'\right)B + 2\tilde{v}_{11}\Sigma_B^{11}\right]^{-1}\left[U' + B'\left(V + \tilde{V}'\right)A + 2\tilde{v}_{11}\Sigma_{AB}^{11}{}'\right]$$

where Σ_{AB}^{ij} is the matrix of covariance of the i_t -th block of A_{t+1} with the j -th block of B_{t+1} and $\tilde{v}_{ij}$ is the (i, j) -th block of $\tilde{V}$.

At time t, y_{t+1} and p_{t+1} are the only stochastic variables in x_{t+1} , and these are supposed to be independent each other, the only non zero entries of $\Sigma_{t+1|t}$ are the matrixes $\Sigma_{t+1|t}^{11}$, $\Sigma_{t+1|t}^{22}$.

The matrix $\tilde{V}$ is given by the Ricatti equation[3]:

$$\tilde{V} = \tilde{Q} + f'R\tilde{f} + 2\tilde{f}'U' + \tilde{f}\left(\tilde{A} + Bf\right)' V \left(A + Bf\right) + \tilde{f}\,\tilde{v}_{11}\left(\tilde{\Sigma}_A^{11} + 2\tilde{\Sigma}_{AB}^{11} f + f'\Sigma_B^{11}\tilde{f}\right) + f\tilde{v}_{22}\Sigma_A^{22} \quad (16)$$

Thus, the variance-covariance matrix depends on the state of the economy x_t , the instruments i_t and the parameters variance as well as those of additive errors. Optimal monetary policy will therefore minimize not only the future gap of the expected state variables to their targets but also their variances condition. Therefore, the optimal policy depends crucially on the degree of uncertainty of the economy. The optimal decision rule is determined by the short-run interest rate as a linear function of the state vector in each period.

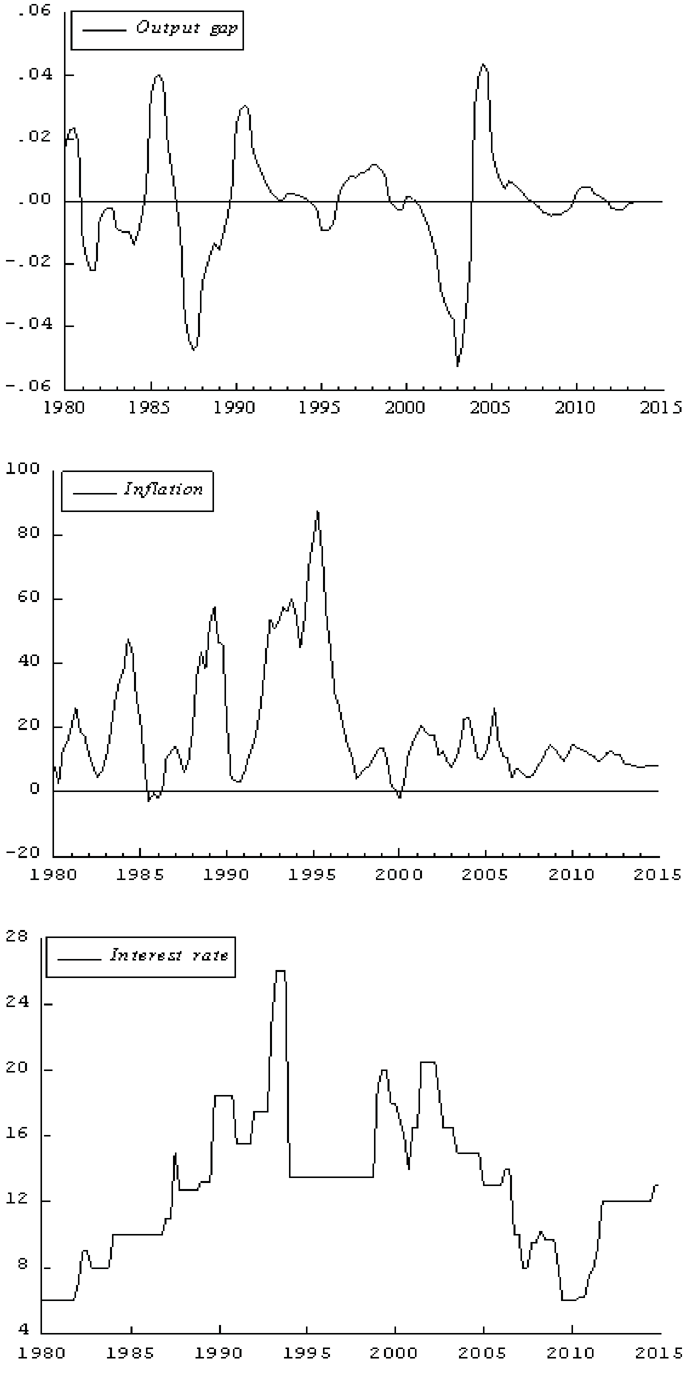
3.4. Data

The model is estimated on quarterly Nigeria data from 1980Q1 to 2015Q1. The data come from the International Financial Statistics (IFS) database of the International Monetary Fund (IMF) and the Live Data Base (LDB) database of the World Bank. The graphic on series data is shown on Figure 2. The output gap is the deviation of output from its potential level. The inflation is measured from the consumer prices index (CPI). The interest rate considered in this study as the instrument of the monetary policy in Nigeria is the discount rate[4].

3 - Also see Chow (1975) or Sargent (1987).

4 - According to Udom and Yaaba (2015), the CBN adjusts both reserve money and interest rate as monetary policy instruments at the same time.

Figure 2 : Nigerian data series



4. Parameter estimation

The Nigeria economy dynamic is captured by applying the robust Ordinary Least Square (OLS) method or simply the General Least Square (GLS) in order to recover a robust covariance matrix of parameters to the IS and Phillips curves. The estimated equations are as follows (*robust standard deviations* are in parentheses and *robustt-student statistics* are in brackets):

$$y_{t+1} = \underset{\substack{(0.0516282)\\ [17.37]}}{0.8966885}y_t - \underset{\substack{(0.0000285)\\ [1.99]}}{0.0000568}\, i_t - p_t \left(+ \hat{e}^{y}_{t+1}\right) \quad (17)$$

$R^2 = 0.7963 \qquad$ *F-statistic* $(2, 138) = 154.09$

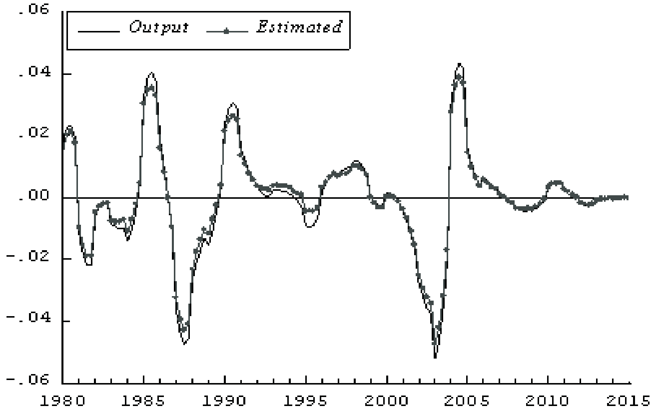
$$p_{t+1} = \underset{\substack{(0.0315267)\\ [30.61]}}{0.9651657}p_t - \underset{\substack{(28.58139)\\ [-1.86]}}{53.23902}y_t + \hat{e}^{p}_{t+1} \quad (18)$$

$R^2 = 0.9417 \qquad$ *F-statistic* $(2, 138) = 486.73$

$s_y^2 = 0.00798 \qquad s_p^2 = 6.5463$

All the coefficients are individually and globally statically significant and have the expected signs except the output coefficient in the Phillips-curve which is also statically significant but high and negative. Nevertheless, the model displays a reasonably good empirical fit of the data for the IS-curve with the R-square equal to 79.63% and for the Phillips-curve with the R-square equal to 94.17% (see also Figures 3 and 4). Although, these estimates suggest a very minor initial role for the CBN's monetary policy, the impact of the lagged value of the output in the Phillips-curve is large implying that the response of the policy rates is much greater in the long-run. The timing assumption of our model is important. Sure enough, at the beginning of each period *t*, the policymakers observe all state variable up to time *t* included. On the basis of those values, the policymakers set the optimal policy rate. Then nominal and real shocks hit the economy, so that at the beginning of period *t*+1 a new vector of state variables influences the central bank's decisions.

Figure 3: Historical and estimated series (Output gap)



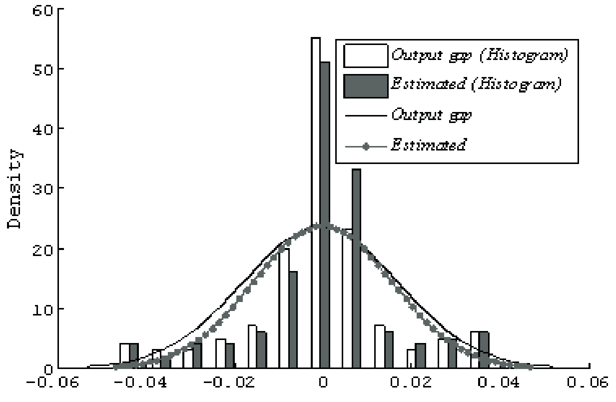
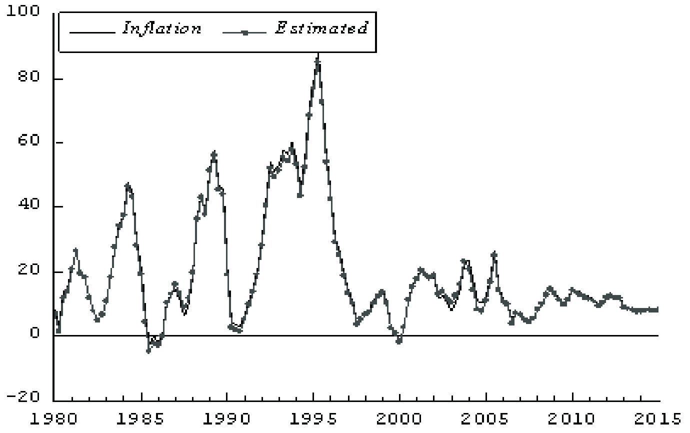
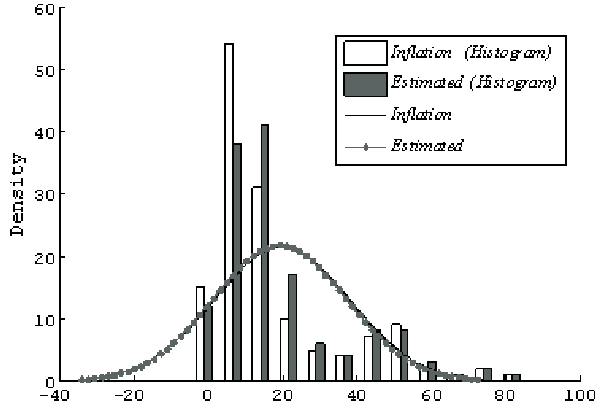


Figure 4: Historical and estimated series (Inflation)





5. The effects of multiplicative parameter uncertainty on optimal policy

The empirical studies which have analyzed the importance of uncertainty on the parameters and the scope of the Brainard conservatism principle generally proceed in two stages: in the first stage a macroeconomic model is estimated and standard deviations of the model parameters are calculated. In a second stage, the optimal policy is calculated under the assumption that the central bank recognizes that the model used is vitiated by uncertainty. For this, the parameters are assumed to be random variables with mean equal to the estimated parameters, and variance equal to the variance empirically estimated. The policymakers are therefore faced within mean to the estimated equations but in fact at each time the coefficients are random values. The selected reaction function is the one that minimizes the expected loss, considering the hazard on the parameters. The resulting values for the means and variances of the stochastic parameters and the values of the non-stochastic parameters are given in the panel of Table 1.

As shown in Table 1, the stochastic parameters come from the estimated model. The discount factor □ is assigned a value of 0.95, implying a discount rate of 5% per period. Since the effects of uncertainty on policy depend crucially on the value of the preference parameters $□_y$ and $□_i$, this

Stochastic parameters			Non -stochastic parameters	
	Mean	Variance		Value
$□_y$	0.8966885	0.0026655	□	0.95
$□_r$	0.0000568	8.123E -10	$□_y$	[0,2]
$□_□$	0.9651657	0.0009939	$□_i$	[0,2]
$□_y$	-53,23902	816,89585		

will be allowed to take values varying from 0, that is, "strict inflation targeting," to 2, with a larger weight on stabilizing output and interest rate than on fighting inflation. In this section, we propose to study successively the initial response of policy, the policy response over time and the implied dynamics of the interest rate.

5.1 The initial response of policy

When the policymakers have an inflation and output stabilization objective, the optimal policy has the following form: $i_t = y_y y_t + y_p p_t$. But when they are concerned to smooth interest rate, the optimal policy rather has the following form: $i_t = y_y y_t + y_p p_t + y_{i_{t-1}} i_{t-1}$. Table 2 provides results of several illustrative cases with different preferences over goal variables.

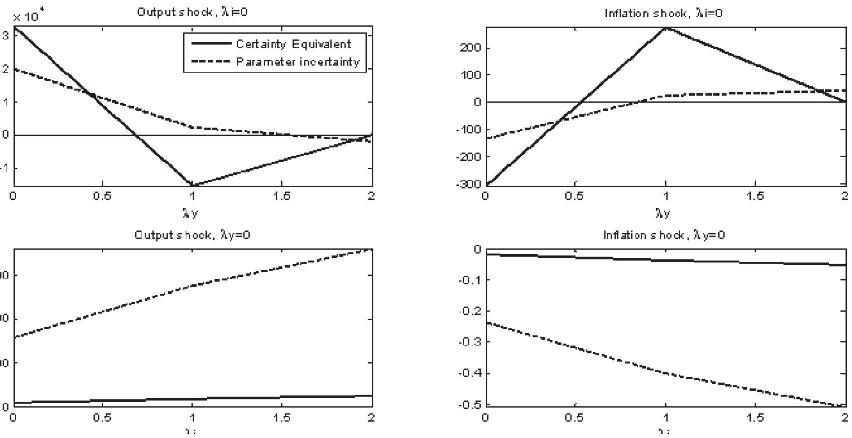
Table 2: Initial responses of interest rate

Loss Fonction / Weight on	$\square_y$	$\square_\square$	$\square_i$
$\square_y = 0$, $\square_i = 0$			
Certainty Equivalence	32779	-307	0
Parameter Uncertainty	20143	-136	0
$\square_y = 0.5$, $\square_i = 0$			
Certainty Equivalence	32776	-307	0
Parameter Uncertainty	20143	-136	0
$\square_y = 0$, $\square_i = 0.5$			
Certainty Equivalence	55.3610	-0.1282	0.8450
Parameter Uncertainty	505.3930	-0.9240	0.6397
$\square_y = 0$, $\square_i = 1$			
Certainty Equivalence	31.1336	-0.0702	0.8779
Parameter Uncertainty	355.0900	-0.6024	0.6849
$\square_y = 1$, $\square_i = 0.5$			
Certainty Equivalence	55.3613	-0.1282	0.8450
Parameter Uncertainty	505.3940	-0.9240	0.6397
$\square_y = 1$, $\square_i = 1$			
Certainty Equivalence	31.1337	-0.0702	0.8779
Parameter Uncertainty	355.0906	-0.6024	0.6849
$\square_y = 0.5$, $\square_i = 0.5$			
Certainty Equivalence	55.3612	-0.1282	0.8450
Parameter Uncertainty	505.3935	-0.9240	0.6397
$\square_y = 0.8$, $\square_i = 0.2$			
Certainty Equivalence	112.3864	-0.2730	0.7954
Parameter Uncertainty	772.6781	-1.5687	0.5808
$\square_y = 0.2$, $\square_i = 0.1$			
Certainty Equivalence	184.0049	-0.4685	0.7542
Parameter Uncertainty	1037.4	-2.3	0.5

When the policymakers have an inflation and output stabilization objective, the weights associated to output gap are relatively far from reality. In the same conditions, the weights associated to output gap and inflation do not seem too much change. The attenuation effect does not coincide well with the accepted wisdom formalized by William Brainard (1967) when the policymakers have only an inflation and output stabilization objective. However, when they are concerned to smooth interest rate, the weights associated especially to inflation and the lagged interest rate are relatively weak under multiplicative parameter uncertainty than under certainty equivalence in accordance with the prediction of the Brainard conservatism principle. It appears as underling it some studies (Sodestrom 1999, Sahuc 2005) that the stem from the policy under multiplicative parameter uncertainty depends on the policymaker's preferences.

The two top graphs of Figure 5 show the initial policy response to current output and inflation shocks for different values of $\square_y$ (when $\square_i$ =0) in the case of certainty equivalence and the case of multiplicative parameter uncertainty, that is, when there is uncertainty about all four parameters of the benchmark model. However, the two bottom graphs of Figure 5 show the initial policy response to current output and inflation shocks for different values of $\square_i$ (when $\square_y$= 0) in the same case as the precedent. For the two top graphs, we have two different possibilities: when $\square_y$ is low, optimal policy responses under parameter uncertainty is more cautious than under certainty equivalence. As $\square_y$ increase, the response under parameter uncertainty is stronger than under certainty equivalence. For the two bottom graphs, when $\square_y$ is low and high, optimal policy responses under parameter uncertainty is stronger than under certainty equivalence. Since at first glance the above results may be counterintuitive, they may need some further explanation. The model used here differs from that of Brainard (1967) in the fact that it is dynamic rather than static in one hand, and it incorporates uncertainty concerning not only the impact effect of policy, but also concerning the dynamic development of the economy on the other hand.

Figuer 5: Reaction fonction coefficients, all parametres uncertain

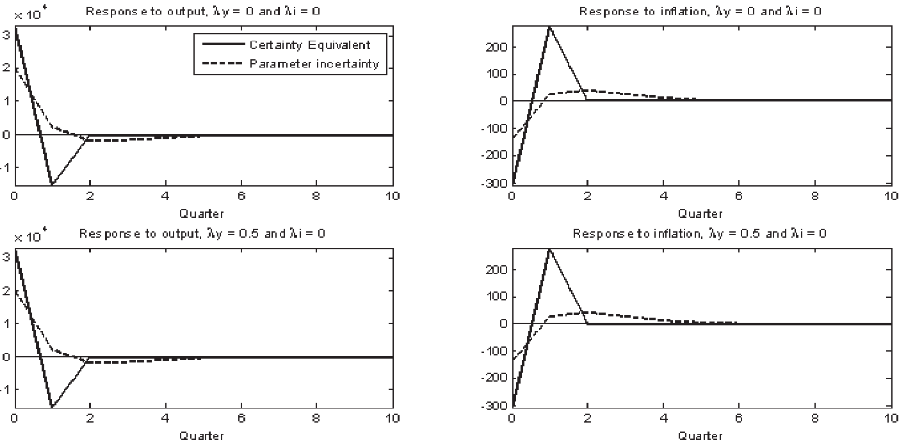


5.2 The policy response over time

The analysis of optimal monetary policy under multiplicative parameter uncertainty also has interesting implications on the dynamic responses of monetary policy, that is, the response of policy to past shocks to output (or demand) and inflation (or supply).

Figure 6 shows the optimal monetary policy responses to supply and demand shocks over the first ten periods following a shock when policymakers are not concerned to smooth interest rate, that is, when they have only an inflation and/or output stabilization objective. The two top graphs of Figure 6 illustrate the case where the central bank preferences are λ_y=0 and λ_i=0 following respectively a demand and supply shocks. However, the two bottom ones illustrate the case where the central bank preferences are λ_y=0.5 and λ_i=0 also following respectively a demand and supply shocks. Figure 6 shows that the response of monetary policy over time varies substantially with the policymaker's preferences λ_y and λ_i. Specifically, for small values of λ_y and λ_i, the optimal policy response to a demand and supply shock under certain parameter configurations is to raise the interest rate instrument in the first period, but then lower it below the initial level and move gradually back to neutral policy. This is shown by the solid lines in the four graphs of Figure 6. In accordance with the intuition of Brainard conservatism principle the central bank behavior is cautious under parameters uncertainty. In this condition, uncertainty about the impact effect of policy still leads to less aggressive policy.

Figuer 6: Reaction fonction coeficients, all parametres uncertain ($\square_i$=0)



Figuer 7: Reaction fonction coeficients, all parametres uncertain ($\square_i$>0)

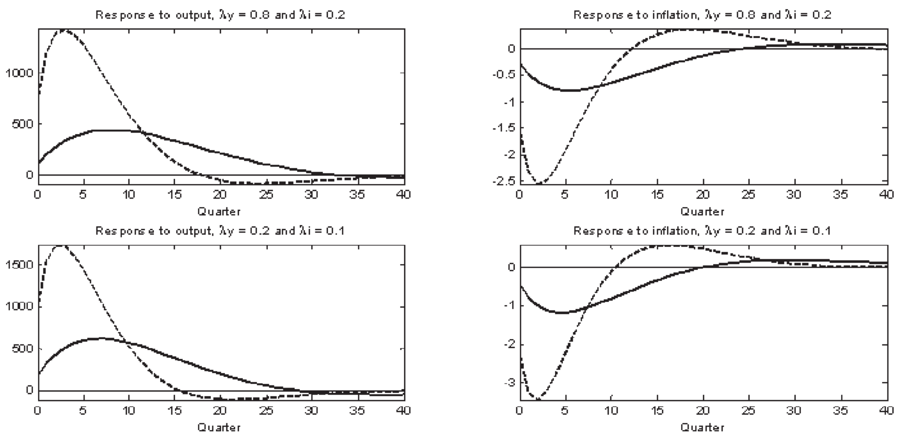


Figure 7 shows the optimal monetary policy responses to supply and demand shocks over the first forty periods following a shock when policymakers are concerned to smooth interest rate, that is, when they have an inflation, output and interest rate stabilization objective. With a more realistic central bank preferences $\square_y$=0.8 and $\square_i$=0.2 (the two top graphs of Figure 7) or $\square_y$=0.2 and $\square_i$=0.1 (the two bottom graphs of Figure 7), the policy responses are more intuitively attractive, since it is implies naturally more interest rate smoothing, in the sense that the initial policy response is more aggressive under uncertainty, policy in later periods is closer to neutral, since the strong initial move has neutralized a larger part of the shock. This is shown by the dotted lines in the four graphs of Figure 7.Thus, when the policymakers are anxious to smooth the interest rate ($\square_i>0$), the policy response, in the first period, is more persistent under multiplicative parameter uncertainty than under certainty equivalence but less persistent in the later periods. Contrary to the intuition of Brainard conservatism principle the central bank behavior is rather aggressive under parameters uncertainty. In this condition, when the policymakers are uncertain about the dynamics of the economy, they might find their optimal to move more aggressively in response to shocks, so as to avoid bad outcomes in the future.

5. CONCLUSION AND RECOMMENDATIONS

5.1 Study conclusion

Monetary policy is inevitably made in an environment of substantial uncertainty and central banks, in particular CBN, have limited knowledge about the structure and functioning of the economy.

This paper looks at the implications of multiplicative parameter uncertainty for the conduct of monetary policy in Nigeria over the period 1980Q1–20105Q1. Theoretically, the certainty equivalence principle indicates that the optimal policy is not affected by the size of uncertainty called "additive". However, the "Brainard conservatism principle" states that under uncertainty about the transmission mechanism, monetary policy should be less aggressive than in certainty universe.

We show in this study that the Brainard principle can be challenged not only by the choice of the model used but also by the preferences of the central bank. Using the linear quadratic stochastic control approach by introduction the variance of estimated parameters in the optimal control theory it is shown with the help of a simple dynamic macroeconomic model that uncertainty about structural parameters does not necessarily lead to more cautious monetary policy, refining the accepted wisdom concerning the effects of multiplicative parameter uncertainty on optimal policy. Specifically, the results through the optimal monetary policy rule yield that the policymakers are always very cautious when they have an inflation and output stabilization objective. However, introducing an interest rate smoothing in the loss function makes the central banker more aggressive.

In sum, while many empirical studies update a "Brainard effect", this study highlights the lack of consensus as on the existence of this effect and on its size. It is likely insufficient to fully account for gradualism observed in practice.

5.2 Policy recommendations

It emerges from this study that parameter uncertainty can induce greater policy activism, that is tosay stronger reaction of nominal interest rate to fluctuations in the rate of inflation and output as against the period of certainty. Thus, when the CBN policymakers have not an interest rate smoothing objective, there is no worry for the stabilization of the monetary policy instrument, in accordance whit the common belief that monetary policy should be less responsive in case of parameter uncertainty, they modify less vigorously interest rate in order to stabilize the economy more quickly after a shock. However, when they choose to smooth interest rates; they modify more vigorously the policy rate in order to stabilize the economy more quickly after a shock.

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APPENDIX

Appendix A: Matrixes representations

The state spaces form is as indicated in equation (12):

$$x_{t+1} = A_{t+1}x_t + B_{t+1}i_t + e_{t+1}$$

where the parameters matrices A_{t+1} and B_{t+1} are stochastic with means

$$A = \begin{bmatrix} g_y & g_r & 0 \\ b_p & b_y & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} -\ g_r \\ 0 \\ 0 \end{bmatrix}$$

and variance-covariance matrices

$$\Sigma_A = \begin{bmatrix} s_{g_y}^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & s_{g_r}^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & s_{b_y}^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & s_{b_p}^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \Sigma_B = \begin{bmatrix} s_{g_r}^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \Sigma_{AB} = \begin{bmatrix} 0 & 0 & 0 \\ -\ s_{g_r}^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Appendix B: Solving the control problem

The central bank solves the problem:

$$J(x_t) = \min_{\{i_t\}} \left\{ (C_x x_t + C_i i_t)' K (C_x x_t + C_i i_t) + f E_t J(x_{t+1}) \right\} \tag{13}$$

under constraint of equation (12):

$$x_{t+1} = A_{t+1}x_t + B_{t+1}i_t + e_{t+1}$$

As the objective function is quadratic and the constraint linear, the value function would have the form:

$$J(x_t) = x_t'\tilde{V}x_t + w \tag{20}$$

The expected value of the state spaces vector is:

$$\mathrm{E}_t x_{t+1} = Ax_t + Bi_t \tag{21}$$

and the matrix of variance-covariance is:

$$\Sigma_{t+1|t} = \begin{bmatrix} \Sigma^{y}_{t+1|t} & \Sigma^{y,p}_{t+1|t} \\ \Sigma^{p,y}_{t+1|t} & \Sigma^{p}_{t+1|t} \end{bmatrix} \tag{22}$$

Since all parameters are assumed to be independent (as proposed by Brainard (1967)[5]), the elements that are not on the diagonal are zero (zero)).

The diagonal elements are:

$$\begin{aligned} \Sigma^{y}_{t+1|t} &= var_t\left(a_y y_t - a_r\right)i_t - p_t\left(+ e^{y}_{t+1}\right) \\ &= x_t'\,\Sigma^{11}_{A}x_t + 2x_t'\,\Sigma^{11}_{AB}i_t + i_t'\Sigma^{11}_{B}i_t + \Sigma^{11}_{e} \end{aligned} \tag{23}$$

and

$$\begin{aligned} \Sigma^{p}_{t+1|t} &= var_t\left(b_p p_t + b_y y_t + e^{p}_{t+1}\right) \\ &= x_t'\,\Sigma^{22}_{A}x_t + \Sigma^{22}_{e} \end{aligned} \tag{24}$$

where Σ^{ij}_{AB} is the matrix of covariance of the i_t-th block of A with the j-th block of B,

$$\Sigma^{11}_{A} = \begin{bmatrix} s^2_{a_y} & 0 & 0 \\ 0 & s^2_{a_r} & 0 \\ 0 & 0 & 0 \end{bmatrix}, \; \Sigma^{22}_{A} = \begin{bmatrix} s^2_{b_y} & 0 & 0 \\ 0 & s^2_{b_p} & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{25}$$

5 - Brainard (1967) did the hypothesis that the covariance between the parameters is zero. However, as Brainard remarks it, the cutting and the sign of the covariance have some implications to the optimum policy. Integrating of the covariance, one can obtain some results opposed in terms of behaviour of the monetary authorities.

$$\Sigma_B^{11} = s_{a_r}^2,\ \Sigma_{AB}^{11} = \begin{bmatrix} 0 \\ -\ s_{a_r}^2 \\ 0 \end{bmatrix} \tag{26}$$

$$\Sigma_e^{11} = s_y^2,\ \Sigma_e^{22} = s_p^2 \tag{27}$$

In consequence, the extra term to take into account in equation (14) is then:

$$tr\left(\Sigma_{t+1|t}\right) = \tilde{v}_{11}\left(x_t'\Sigma_A^{11}x_t + 2x_t'\Sigma_{AB}^{11}i_t + i_t'\Sigma_B^{11}i_t + \Sigma_e^{11}\right) + \tilde{v}_{22}\left(x_t'\Sigma_A^{22}x_t + \Sigma_e^{22}\right) \tag{28}$$

where $\tilde{v}_{11}$ and $\tilde{v}_{33}$ are the elements of the diagonal of the matrix $\tilde{V}$,

Using (22)-(23) in (16), the Bellman equation is:

$$x_t'\tilde{V}x_t + \tilde{w} = \min_{\{i_t\}}\left\{\left(C_x x_t + C_i i_t\right)' K\left(C_x x_t + C_i i_t\right) + f\left(Ax_t + Bi_t\left(\tilde{V}\right)Ax_t + Bi_t\right) + f\,tr\left(\tilde{V}\Sigma_{t+1|t}\right) + f\,\tilde{w}\right\}$$

(29)

which gives the necessary first order condition as follows:

$$f\left\{\left[U' + B'\left(\tilde{V} + \tilde{V}'\right)A\right]x_t + \left[R + B'\left(\tilde{V} + \tilde{V}'\right)B\right]i_t + \frac{dtr\left(\tilde{V}\Sigma_{t+1|t}\right)}{di_t} + f\,\tilde{w}\right\} = 0 \tag{30}$$

where from equation (30) we have:

$$\frac{dtr\left(\tilde{V}\Sigma_{t+1|t}\right)}{di_t} = 2\tilde{v}_{11}\left(\Sigma_{AB}^{11}{}'x_t + \Sigma_B^{11}i_t\right) \tag{31}$$

Like this, we obtain the optimal policy:

$$i_t = - \left[R + B'\left(\tilde{V} + \tilde{V}'\right)B + 2\tilde{v}_{11}\Sigma_B^{11}\right]^{-1}\left[U' + B'\left(\tilde{V} + \tilde{V}'\right)A + 2\tilde{v}_{11}\Sigma_{AB}^{11}{}'\right]x_t$$

$$= \tilde{f}x_t$$

Finally, using the equation (21) and the policy rule (32) in the equation (29) of Bellman, one obtains:

$$x_t'\tilde{V}x_t + \tilde{w} = \left(C_x x_t + C_i i_t\right)' K \left(C_x x_t + C_i i_t\right) + f\left[\left(Ax_t + Bfx_t\right)'\tilde{V}\left(Ax_t + Bfx_t\right)\right]$$

$$+ f\tilde{v}_{11}\left(x_t'\Sigma_A^{11}x_t + 2x_t'\Sigma_{AB}^{11}fx_t + x_t'f'\Sigma_B^{11}fx_t + \Sigma_e^{11}\right) \qquad (33)$$

$$+ f\tilde{v}_{22}\left(x_t'\Sigma_A^{22}x_t + \Sigma_e^{22}\right),$$

Then the matrix is determined by:

$$\tilde{V} = \tilde{Q} + f'\tilde{R}f + 2f'U' + f\left(A + B\tilde{f}\right)'V\left(\tilde{A} + B\tilde{f}\right) + f\tilde{v}_{11}\left(\Sigma_A^{11} + \tilde{2}\Sigma_{AB}^{11}f + \tilde{f}'\Sigma_B^{11}\tilde{f}\right) + f\tilde{v}_{22}\Sigma_A^{22}. \qquad (34)$$